Worksheet for Sections 9.4 and 9.5

1. (a) Show that $\dfrac{2}{(n+1)(n+3)} = \dfrac{1}{n+1} - \dfrac{1}{n+3}$ for all $n \geq 1$.

   (b) Write out the 5th partial sum of $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+3} \right)$. Then find its numerical value.

   (c) Find the sums of the series $\displaystyle\sum_{n=1}^{\infty} \frac{2}{(n+1)(n+3)}$ and $\displaystyle\sum_{n=12}^{\infty} \frac{2}{(n+1)(n+3)}$.

2. (a) Write $0.636363\cdots$ as a geometric series, and find its numerical value. (*Hint:* $0.636363\cdots\cdots = 0.63 + .0063 + .000063 + \cdots$.)

   (b) Write $0.999\cdots$ as a geometric series, and find its numerical value.

   (c) Does (b) indicate that $0.999\cdots = 1$? Discuss with your group.

3. When a superball is dropped onto a hardwood floor, it bounces up to approximately 80% of its original height. Suppose a superball is dropped from 5 feet above a hardwood floor.

   (a) Find the distance (in feet) that the ball travels during three bounces (that is, until it reaches its maximum height after bouncing on the floor three times).

   (b) Find a formula involving a geometric series for the distance the ball travels (in theory) if it is left to bounce forever. Then evaluate the sum of the series you found.

4. From Exercise 48(a) on p. 602, we know that the numerical value of $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ is $\dfrac{\pi^2}{6}$.

   (a) Using this information, explain in a complete sentence why we can, or cannot, conclude that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1.9}}$ converges. If $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1.9}}$ converges, what are the relative sizes of $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1.9}}$ and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$?

   (b) Similarly, explain in a complete sentence why we can, or cannot, conclude that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{2.1}}$ converges. If $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{2.1}}$ converges, what are the relative sizes of $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{2.1}}$ and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$?